Answer Key

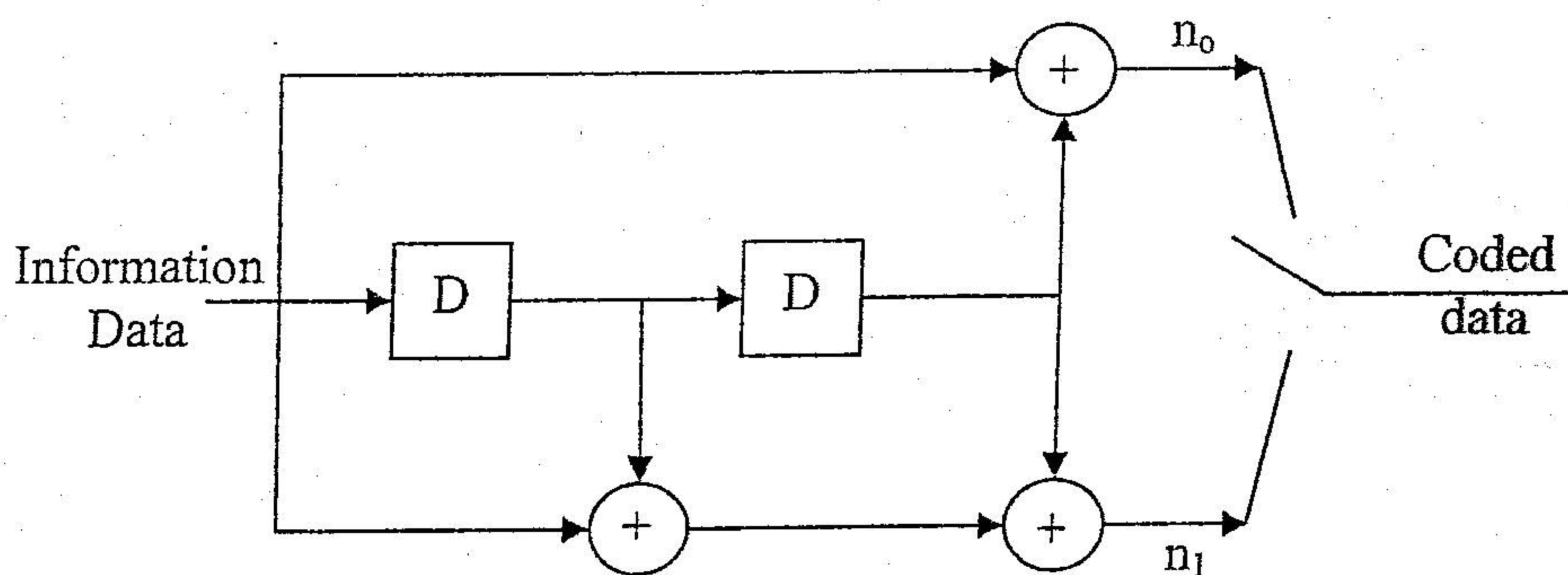
نظرية المعلومات والترميز - نهائي

Specialization:	Electrical Engineering		Palestinian National Authority Ministry Education & Higher Education Palestine Technical University College of Engineering & Technology
Course Name:	Information Theory and Coding		
Date:	28/05/2011		
Time:	8:30-10:30		Final Exam
Instructor:	Dr. Mutamed Khatib		Second semester 2010/2011

Answer *all* the following 5 questions

Q1. *(8 marks)* Find the entropy, redundancy and information rate for a four-symbol source (A, B, C, D) with a baud rate of 1024 symbols/s and symbol selection probabilities of 0.2, 0.5, 0.2, 0.1 under the condition that the source has one-symbol memory location such that no two consecutively selected symbols can be the same.

Q2. *(8 marks)* Encode the sequence 1000 (I_o I_1 I_2 I_3) using the rate ½ constraint length 3 binary (2,1,3) convolutional code, with the encoder below:



Q3. *(8 marks)* Use Public Key Encryption method to encrypt the plane-text message number 21. Choose the smallest possible values of *P* and *Q*. Decipher the resulting cipher-text to check your answer

Q4. *(8 marks)* Consider the random variable

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ 0.5 & 0.26 & 0.11 & 0.04 & 0.04 & 0.03 & 0.02 \end{pmatrix}$$

(a) Find a binary Huffman code for *X*:

(b) Find the expected code length for this encoding.

Q5 *(8 marks)* Given a ½ rate convolutional encoder defined by $P_1(x)=1+x+x^2$ and $P_2(x)=1+x^2$, and assuming data is fed into the shift register 1 bit at a time, draw the encoder tree diagram.

Good luck

تم الرفع بواسطة
م. معن أبو عيسى

Q.1 $P(A|A) = P(B|B) = P(C|C) = P(D|D) = 0$

$P(B) = 0.5 \Rightarrow P(\bar{B}|B) = P(B|\bar{B}) = 1$ $\quad \bar{B}$: not B

$\Rightarrow P(A|B) + P(C|B) + P(D|B) = 1$

$P(B|A) + P(B|C) + P(B|D) = 1$

$P(A|C) = P(A|D) = P(C|A) = P(C|D) = P(D|A) = P(D|C) = 0$

$P(A) = P(A|A)P(A) + P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D)$

$0.2 = 0 + P(A|B)\,0.5 + 0 + 0$

$\Rightarrow P(A|B) = \frac{0.2}{0.5} = 0.4$

$P(C) = P(C|A)P(A) + P(C|B)P(B) + P(C|C)P(C) + P(C|D)P(D)$

$0.2 = 0 + P(C|B)\,0.5 + 0 + 0$

$\Rightarrow P(C|B) = \frac{0.2}{0.5} = 0.4$

$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) + P(D|D)P(D)$

$0.1 = 0 + P(D|B)\,0.5 + 0 + 0$

$\Rightarrow P(D|B) = \frac{0.1}{0.5} = 0.2$

$H = \sum_i P(i) \sum_j P(j|i) \log_2 \frac{1}{P(j|i)}$ bit/symbol.

$= P(A)\left[P(A|A)\log_2\frac{1}{P(A|A)} + P(B|A)\log_2\frac{1}{P(B|A)} + P(C|A)\log_2\frac{1}{P(C|A)} + P(D|A)\log_2\frac{1}{P(D|A)}\right]$

$+ P(B)\left[P(A|B)\log_2\frac{1}{P(A|B)} + P(B|B)\log_2\frac{1}{P(B|B)} + P(C|B)\log_2\frac{1}{P(C|B)} + P(D|B)\log_2\frac{1}{P(D|B)}\right]$

$+ P(C)\left[P(A|C)\log_2\frac{1}{P(A|C)} + P(B|C)\log_2\frac{1}{P(B|C)} + P(C|C)\log_2\frac{1}{P(C|C)} + P(D|C)\log_2\frac{1}{P(D|C)}\right]$

$+ P(D)\left[P(A|D)\log_2\frac{1}{P(A|D)} + P(B|D)\log_2\frac{1}{P(B|D)} + P(C|D)\log_2\frac{1}{P(C|D)} + P(D|D)\log_2\frac{1}{P(D|D)}\right]$

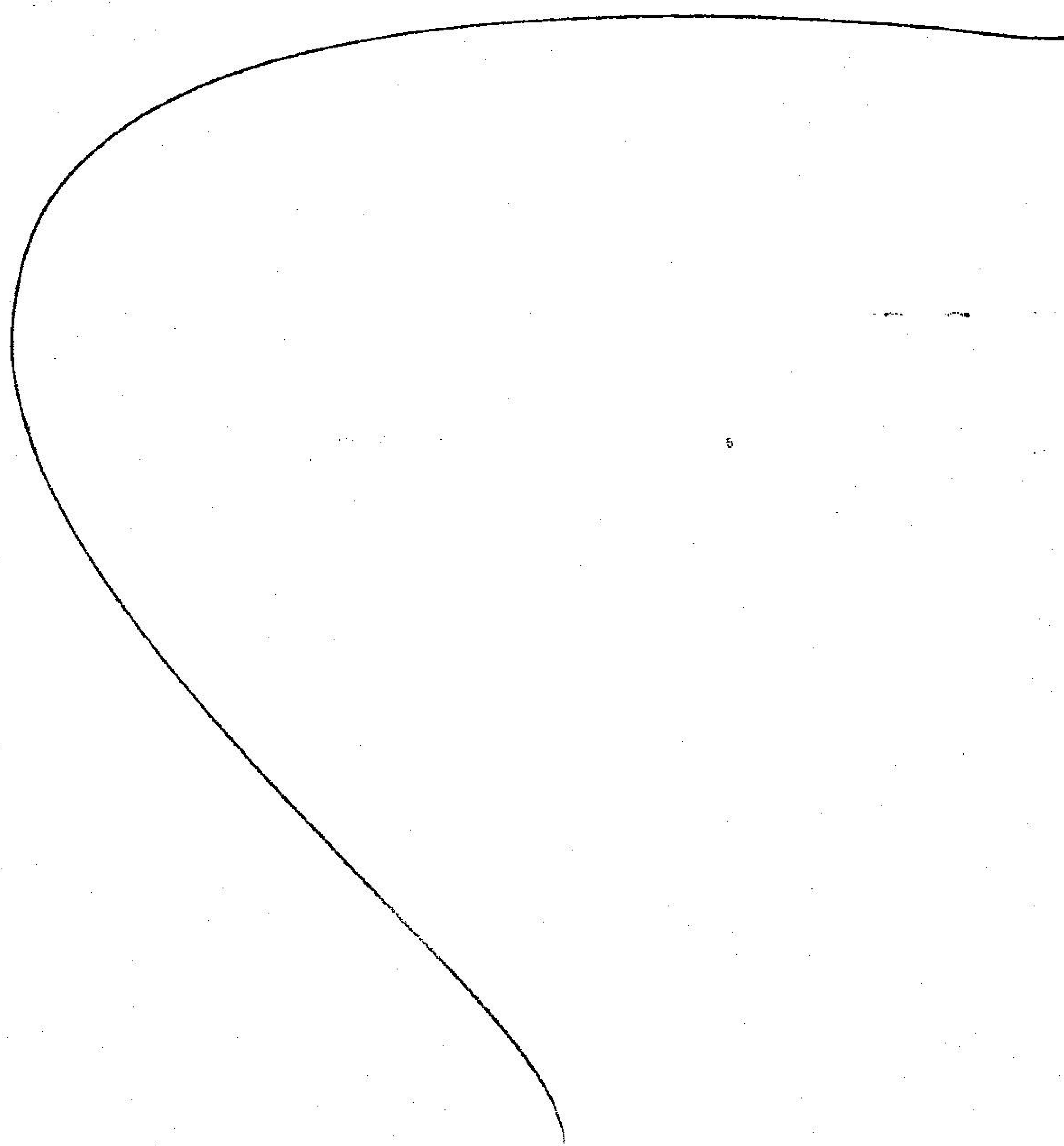
$$\Rightarrow H = 0.2\left[0 + 1\log_2\frac{1}{1} + 0 + 0\right] + 0.5\left[0.4\log_2\frac{1}{0.4} + 0 + 0.4\log_2\frac{1}{0.4} + 0.2\log_2 \ldots\right.$$

$$+ 0.2\left[0 + 1\log 1 + 0 + 0\right] + 0.1\left[0 + 1\log_2 1 + 0 + 0\right]$$

$$= 0.761 \text{ bit/symbol.}$$

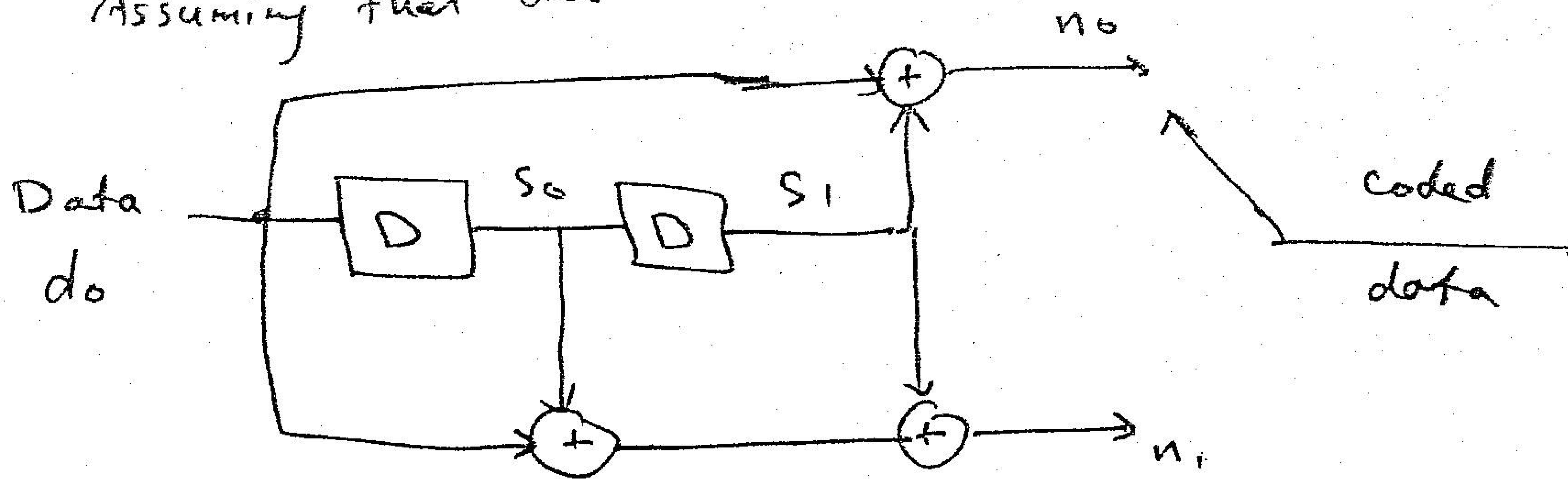
$$R = H_{max} - H = \log_2 4 - 0.761 = 1.239 \text{ bit/symbol.}$$

$$R_i = R_s H = 1024 \times 0.761 = 779 \text{ bit/s.}$$



Q.2 There is only one data bit per block; and hence no need for an input demultiplexer, and two code bits, generated by the 2 sets of modulo 2 adders shown.

Assuming that the encoder starts from zero state:



do	So	S1	no	n1
1	0	0	1	1
0	1	0	0	1
0	0	1	1	1
0	0	0	0	0

⇒ coded data 11011100

Q.3 $T = 2$ $\quad C = T^N \bmod N$

$N > T$

$N = PQ \Rightarrow P = 5 ; Q = 7 \Rightarrow N = 5 \times 7 = 35$

$C = 2^{35} \bmod 35 = 18$

to decipher :

$T = C^{P'} \bmod Q$ where $PP' = 1 \bmod (Q-1)$

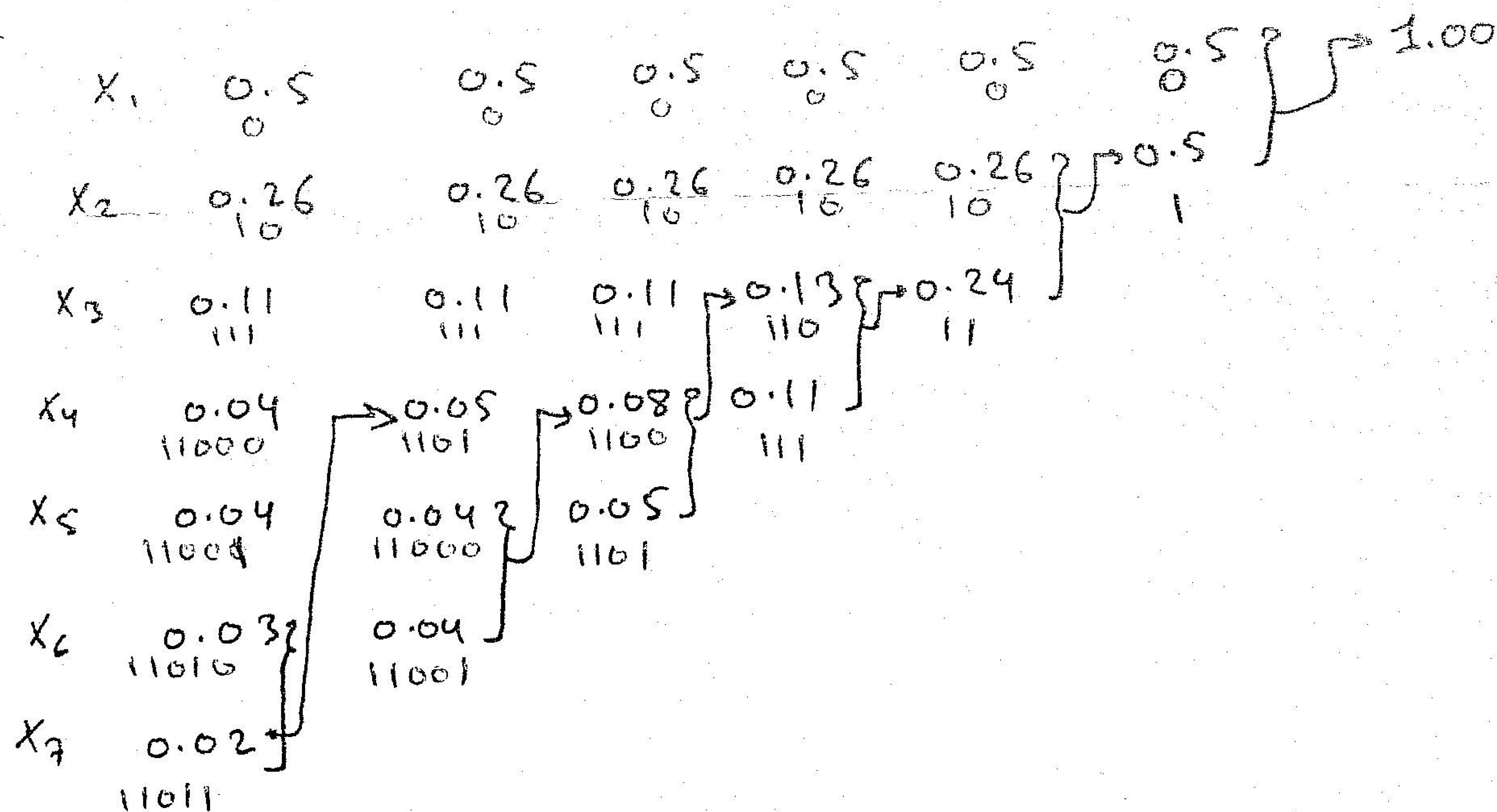
$\Rightarrow 5P' = 1 (\bmod 6) \Rightarrow 5P' = 7, 13, 19, 25, 31,$

x x x ✓

$\Rightarrow P' = 5$

$\Rightarrow T = 18^5 \bmod 7 = 2$

Q.4



	x_1	x_2	x_3	x_4	x_5	x_6	x_7
ode	0	10	111	11000	11001	11010	11011
L	1	2	3	5	5	5	5
	0.5	0.26	0.11	0.04	0.04	0.03	0.02

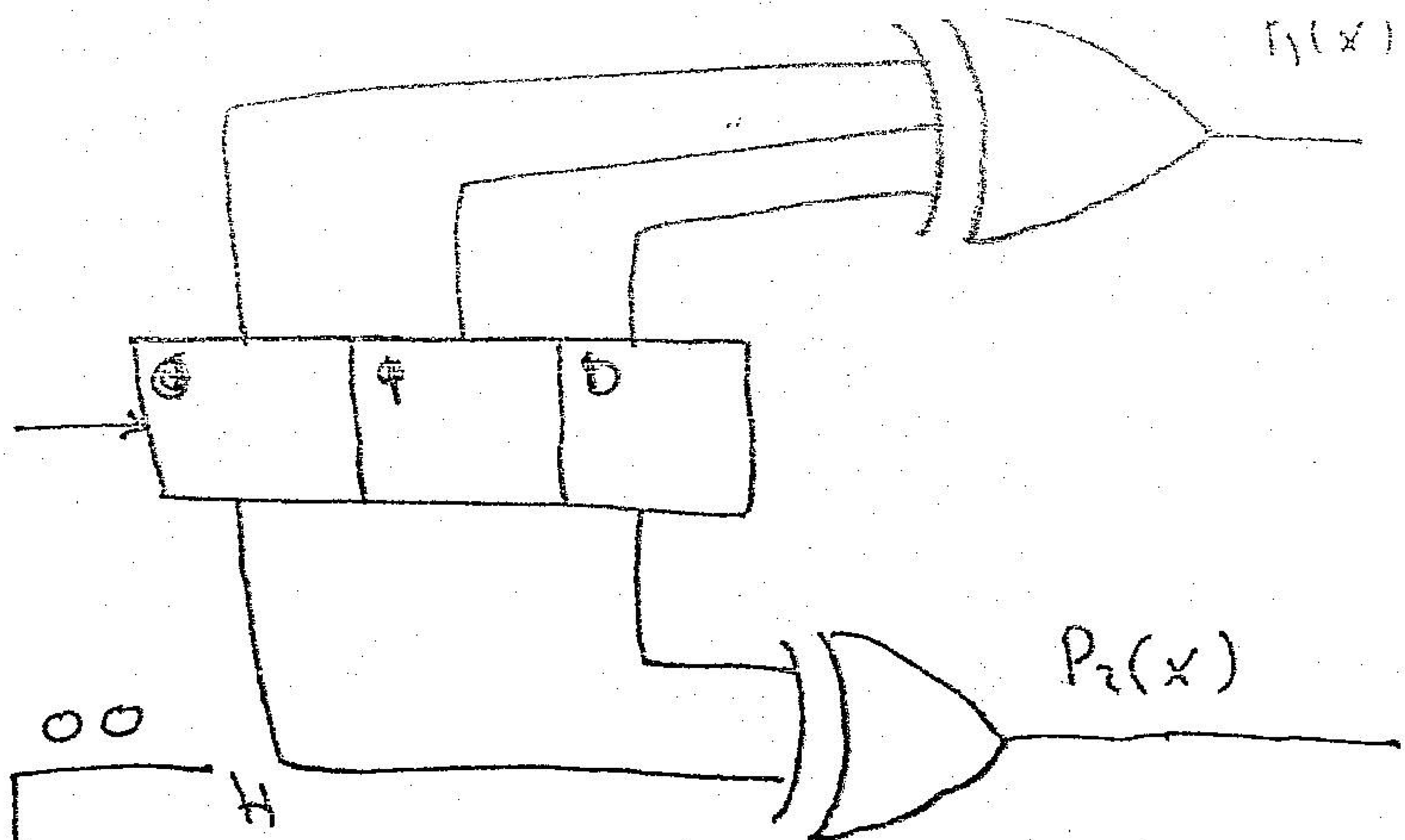
$H = 0.5 \times 1 + 0.26 \times 2 + 0.11 \times 3 + 0.04 \times 5 + 0.04 \times 5 + 0.03 \times 5 + 0.02 \times 5$

$= 2$

$\Rightarrow$ Expected Length is 2

Q.5 $P_1(x) = 1 + x + x^2$

$P_2(x) = 1 + x^2$



Tree diagram

